

Light-quark contributions to the magnetic form factor of the $\Lambda(1405)$

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In a recent study of the $\Lambda(1405)$, the suppression of the strange-quark contribution to the magnetic form factor was interpreted as the discovery of a dominant antikaon-nucleon composition for this low-lying state. We confirm this result by calculating the light u - and d -quark contributions to the $\Lambda(1405)$ magnetic form factor in lattice QCD in order to determine the extent to which their contributions support this exotic molecular description. Drawing on the recent graded-symmetry approach for the flavor-singlet components of the $\Lambda(1405)$, the separation of connected and disconnected contributions is performed in both the flavor-octet and singlet representations. The relationship between light-quark contributions to the $\Lambda(1405)$ magnetic form factor and the connected contributions of the nucleon magnetic form factors is established and compared with lattice calculations of the same quantities, confirming the $\bar{K}N$ molecular structure of the $\Lambda(1405)$ in lattice QCD.

Resolving and understanding the internal structure of hadronic excited states is an important contemporary problem in the field of nonperturbative QCD. While lattice QCD simulation methods are increasingly able to probe the chiral regime of ground state observables with unprecedented accuracy [1–4], the resolution of excited-baryon form factors is still at a very early stage [5–10].

Interest in the $\Lambda(1405)$ resonance has continued unabated for more than 50 years [10–44] because of its unusually low mass – lower even than the corresponding mass of the negative parity nucleon, despite containing a heavier strange quark. The unexpected position of the $\Lambda(1405)$ in the spectrum has been explored in several studies, which typically indicate a significant contribution from a $\bar{K}N$ bound state [10–39]. The $\pi\Sigma$ channel also plays a nontrivial role. It is now widely agreed that there is a two-pole structure in this resonance region [18–32] stemming from attractive interactions in both the $\pi\Sigma$ and $\bar{K}N$ channels. In making contact with results from lattice QCD [10, 40, 41], a description of the $\Lambda(1405)$ over a range of quark masses has been developed [10, 32, 45], bridging constituent-quark ideas at heavy quark masses and the molecular $\bar{K}N$ dominance of the $\Lambda(1405)$ at light quark masses.

A recent lattice QCD study of the $\Lambda(1405)$ reported evidence of a molecular $\bar{K}N$ structure [10]. There, the role of the strange quark was paramount in signaling the presence of a dominant $\bar{K}N$ structure. At heavier quark masses approaching the strange quark mass, the three quark flavors (u , d , s) are found to make approximately equal contributions to the magnetic form factor when their charges are set to unity. The underlying flavor symmetry is manifest. However, as the u and d quarks become light, flavor symmetry in the quark contributions to the magnetic form factor is found to be badly broken, and the strange-quark contribution drops by an order of magnitude from its maximum to a nearly vanishing value at the smallest quark mass.

This feature has a simple explanation in terms of a $\bar{K}N$ molecule. The strange quark is confined in a spin-0 kaon in a relative S wave about the nucleon, implying a net absence of angular momentum. Hence, the strange quark cannot contribute to the magnetic form factor of a $\Lambda(1405)$ composed as a molecular $\bar{K}N$ bound state.

In this Letter we focus on the light-quark sector of the magnetic form factor of the $\Lambda(1405)$ in lattice QCD. Until now, it has received little attention. Nevertheless, it is a vital piece of information in the quest to confirm whether the lattice QCD value supports the $\bar{K}N$ molecular description, and is complementary to the strange sector analysis of Ref. [10].

The analysis of the light-quark sector is not straightforward. Careful attention must be given to what has (and has not) been included in the lattice QCD calculation. In particular, the calculations so far [10] omit photon couplings to quark–antiquark loops in the vacuum, which in turn interact with the connected quarks via gluon exchange. These so-called disconnected loop contributions are unlikely to be determined in the near future because of the difficulty they present in numerical simulations of baryon excited states. As the loop is correlated with the quarks carrying the quantum numbers of the state only via gluon exchange, resolving a nontrivial signal requires high statistics and innovative methods. While there has been recent success in isolating the relevant disconnected-loop contributions in ground-state baryon matrix elements [3, 4], challenges in isolating baryon excitations in lattice QCD [10, 40, 45–57] render the resolution of disconnected contributions elusive.

Here, we draw on partially-quenched chiral effective field theory [44, 58–66] to understand the relative weight of these disconnected contributions to the form factors in QCD. With this insight, one can test quantitatively whether the light-quark contribution to the magnetic form factor of the $\Lambda(1405)$, calculated in lattice QCD, is consistent with a molecular $\bar{K}N$ description of the internal structure.

In the $\bar{K}N$ picture, the spin-0 kaon is in a relative S wave about the nucleon. Therefore the light-quark contributions to the magnetic form factor of the $\Lambda(1405)$ have their origin solely in the magnetic form factors of the nucleon. As the couplings for $\Lambda^* \rightarrow K^- p$ and $\Lambda^* \rightarrow \bar{K}^0 n$ are equal, the light sector contribution is related to an average of n and p magnetic form factors in full QCD.

To explore this in further detail, consider the following simple model for the $\Lambda(1405)$

$$|\Lambda^*\rangle = \frac{1}{\sqrt{2}} \left(|K^- p\rangle + |\bar{K}^0 n\rangle \right). \quad (1)$$

In full QCD (with disconnected sea-quark loop contributions included), the form of the quark sector contributions to the light-quark magnetic form factor $\mu_q(Q^2)$ is simple:

$$\begin{aligned} \langle \Lambda^* | \hat{\mu}_q | \Lambda^* \rangle &= \frac{1}{2} \langle K^- p | \hat{\mu}_q | K^- p \rangle + \frac{1}{2} \langle \bar{K}^0 n | \hat{\mu}_q | \bar{K}^0 n \rangle, \\ &= \frac{1}{2} \langle p | \hat{\mu}_q | p \rangle + \frac{1}{2} \langle n | \hat{\mu}_q | n \rangle. \end{aligned} \quad (2)$$

Here the zero spin and relative S wave orbital angular momentum of the kaon about the nucleon has been taken into account. As $m_u = m_d$ in the lattice QCD simulations [10], we consider the charge-symmetric limit of the nucleon magnetic form factors. Since the disconnected sea-quark loop contributions to the magnetic form factor are not accessible for the $\Lambda(1405)$, we neglect them consistently throughout our comparison to the nucleon magnetic form factors.

To make the charge symmetry manifest in our results [67], we work with single quarks of unit charge [68–70], and define the operator $\hat{\mu}_q$, omitting the electric charge factors of $2/3$ and $-1/3$. The doubly- and singly-represented quark sector contributions to the nucleon form factors are defined as $u_p = d_n$ and $d_p = u_n$, respectively, where the subscripts indicate the baryon in which the quark resides. The connected contributions to the nucleon form factors in the charge-symmetric limit are then

$$\langle p | \hat{\mu}_u | p \rangle = 2 u_p, \quad \langle n | \hat{\mu}_u | n \rangle = 1 u_n = 1 d_p, \quad (3)$$

$$\langle p | \hat{\mu}_d | p \rangle = 1 d_p, \quad \langle n | \hat{\mu}_d | n \rangle = 2 d_n = 2 u_p, \quad (4)$$

where the numerical factor counts the quarks. These matrix elements are readily calculated in lattice QCD via the methods introduced in [68].

Returning now to the $\bar{K}N$ picture, Eq. (2) yields a u -quark contribution to the $\Lambda(1405)$ magnetic form factor given by

$$\langle \Lambda^* | \hat{\mu}_u | \Lambda^* \rangle = \frac{1}{2} (2 u_p + d_p), \quad (5)$$

where a proton labeling has been used for $u_n = d_p$. Similarly, the d -quark contribution is

$$\langle \Lambda^* | \hat{\mu}_d | \Lambda^* \rangle = \frac{1}{2} (d_p + 2 u_p). \quad (6)$$

Thus the isospin-symmetry of the $\Lambda(1405)$ is manifest with a light-quark contribution of

$$\langle \Lambda^* | \hat{\mu}_\ell | \Lambda^* \rangle = u_{\Lambda^*} = d_{\Lambda^*} = \frac{1}{2} (2 u_p + d_p). \quad (7)$$

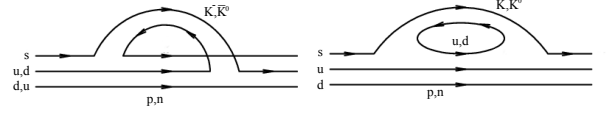


FIG. 1. The leading-order loop contributions from the process: $\Lambda(1405) \rightarrow \bar{K}N$.

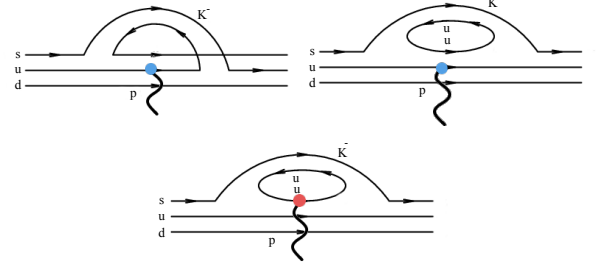


FIG. 2. (color online). The quark flow diagrams for the process $\Lambda(1405) \rightarrow K^- p$ can be decomposed into a completely-connected part and two parts involving disconnected sea-quark loop contributions. The upper-left completely-connected diagram and the upper-right diagram are included in the lattice QCD calculations as the photon couples to a quark flowing in a connected manner from the source to the sink. The case where a photon couples to a disconnected sea quark loop, illustrated in the lower diagram, is not included in the lattice QCD calculations of $\langle \Lambda^* | \hat{\mu}_\ell | \Lambda^* \rangle$.

While we have been careful to omit disconnected sea-quark loop contributions to the nucleon form factors, our simple $\bar{K}N$ model includes an implicit disconnected contribution that has not been included in the lattice QCD calculation of the $\Lambda(1405)$ magnetic form factor. We now identify that contribution, calculate it, and remove it from Eq. (7).

Figure 1 illustrates the connected and disconnected $\bar{K}N$ loop contributions to the two-point function governing the mass of the $\Lambda(1405)$ in full QCD. As the lattice calculations are performed on $2 + 1$ flavor dynamical fermion gauge field configurations, both of these diagrams are included.

The difficulty with sea-quark loop contributions to the magnetic form factor of the $\Lambda(1405)$ is illustrated in Fig. 2, where u_p is considered. Recalling that photon couplings to the spin-0 kaon in a relative S wave about the nucleon do not contribute to the magnetic form factor of the $\Lambda(1405)$, the focus is on the nucleon couplings. In the upper quark-flow diagrams of Fig. 2, the photon couples to u quarks flowing from source to sink. These connected insertions of the photon current are included in the lattice QCD calculations of $\langle \Lambda^* | \hat{\mu}_\ell | \Lambda^* \rangle$.

However, the coupling of the photon to the disconnected sea quark loop, illustrated in the lower diagram of Fig. 2, is not included. As the upper-right and lower diagrams contribute with equal weight, half of the disconnected sector is included, and half is omitted. The task that remains is to understand the relative contributions of the fully-connected diagram and those involving a disconnected sea-quark loop. Thus, we return our attention to Fig. 1.

To determine the relative weight of the couplings between

the connected and disconnected diagrams of Fig. 1, we draw upon partially-quenched chiral perturbation theory [44, 58–66]. For Λ baryons composed of three quark flavors, the graded symmetry approach [44, 58–61] is preferred over the diagrammatic approach [62, 65]. The graded symmetry approach extends standard chiral perturbation theory by introducing commuting ghost field counterparts (\tilde{u} , \tilde{d} , \tilde{s}) to the three quarks (u , d , s). As the ghost-quark fields only enter the interaction through the disconnected loops, they allow the decomposition of the quark flow diagrams into connected and disconnected parts. This can be seen in Fig. 1, where the completely connected diagram contains only valence quarks and the disconnected loop diagram allows sea quarks to contribute to the amplitude. The contributions from the disconnected diagram are isolated by extracting the ghost meson-baryon contribution to each vertex in the diagram.

In the $SU(3)$ -flavor limit, the $\Lambda(1405)$ is identified as the low-lying flavor-singlet baryon. However, as one approaches the physical regime, significant mixing with octet-flavor symmetry is encountered [10, 40]. Therefore one needs to consider both flavor-octet and flavor-singlet couplings for $\Lambda^* \rightarrow \bar{K}N$. In addressing the latter, we draw upon the recently developed graded-symmetry approach for singlet baryons, augmenting the standard octet-baryon Lagrangian with the necessary additional terms [44].

First, we consider the contributions to the singlet component of the $\Lambda(1405)$, denoted Λ'^* , where the prime indicates that a singlet representation is taken, and the star indicates that the resonance has odd parity. In the case of the process $\Lambda'^* \rightarrow K^- p$, the relevant ghost term in the Lagrangian takes the form

$$-g_s \sqrt{\frac{2}{3}} \bar{\Lambda}'^* \tilde{K}^- \tilde{\Lambda}_{p,\tilde{u}}^+, \quad (8)$$

where g_s is taken to be the coupling of the singlet to octet-octet process $\Lambda'^* \rightarrow \pi_0 \Sigma_0$. Here, we follow the notation of Ref. [44]: \tilde{K}^- is composed of a strange quark (s) and a ghost anti-up quark (\tilde{u}) and $\tilde{\Lambda}_{p,\tilde{u}}^+$ represents a proton-like particle composed of $\tilde{u}ud$, with the normal quarks in an anti-symmetric formation. The factor $\sqrt{2/3}$ is derived from the $SU(3|3)$ symmetry relations that govern the Lagrangian.

With reference to the full QCD amplitude,

$$g_s \bar{\Lambda}'^* K^- p, \quad (9)$$

the relative weights of the diagrams in Fig. 1 can be resolved. As a consequence of flavor symmetry, the connected diagram has weight $(1/3)g_s^2$ and the disconnected diagram has weight $(2/3)g_s^2$. Similar results are found for $\Lambda'^* \rightarrow \bar{K}^0 n$, where a d quark participates in the loop in full QCD, such that a comparison with the partially quenched term resolves the same weightings as above.

Significant flavor-symmetry breaking in the physical quark-mass regime admits an important flavor-octet symmetry in the structure of the $\Lambda(1405)$. Thus, one must also consider octet-to-octet meson and baryon contributions. Upon partial quenching, the corresponding couplings derived are

$$\frac{\sqrt{2}(D+3F)}{3} \left\{ \frac{\bar{\Lambda}^* \tilde{K}^- \tilde{\Lambda}_{p,\tilde{u}}^+}{\bar{\Lambda}^* \bar{K}^0 \tilde{\Lambda}_{n,\tilde{d}}^0} \right\} \quad (10)$$

for the u - and d -quark loops, respectively. In full QCD, both the $\Lambda^* \rightarrow K^- p$ and $\Lambda^* \rightarrow \bar{K}^0 n$ channels have the coupling $-(D+3F)/\sqrt{3}$. Thus, one observes the same ratio of $\sqrt{2/3}$ between the disconnected sea-quark loop component couplings and the full QCD couplings. Again, the connected diagram holds a weight of $1/3$ and the disconnected diagram holds a weight of $2/3$ of the full QCD process. As the split between connected and disconnected components is the same for the different flavor representations, the calculation of the partially quenched value of the magnetic form factor is straightforward.

The ratio between the connected and disconnected weights determines the extent to which the full QCD magnetic form factor is changed on the lattice due to the omission of photon couplings to the disconnected sea-quark loops. Consider, for example, the u -quark contribution in the proton, u_p , where the $K^- p$ intermediate state contains a disconnected u -quark contribution. While one-third of the result is preserved in the connected contribution, only half of the remaining two-thirds involving disconnected contributions is preserved. Thus, one can obtain the u -quark contributions to the proton that are included in the lattice QCD calculations by subtracting off $1/2 \times 2/3 = 1/3$ of the full QCD contribution. The u -quark contribution to the neutron, u_n , is fully included in the lattice QCD calculation as, in the $\Lambda'^* \rightarrow \bar{K}^0 n$ channel, the disconnected quark-loop flavor is a d quark, not a u quark, so no adjustment is required. In summary,

$$\langle \Lambda^* | \hat{\mu}_u^{\text{conn}} | \Lambda^* \rangle = \frac{1}{2} \left(\langle K^- p | \hat{\mu}_u | K^- p \rangle - \frac{1}{2} \frac{2}{3} \langle K^- p | \hat{\mu}_u | K^- p \rangle \right) + \frac{1}{2} \langle \bar{K}^0 n | \hat{\mu}_u | \bar{K}^0 n \rangle = \frac{1}{2} \left(2u_p - \frac{2}{3}u_p + u_n \right). \quad (11)$$

The first two terms in the leading parentheses of Eq. (11)

represent the connected u -quark contribution from the proton

component within the $\Lambda(1405)$. The first term provides the full QCD contribution while the second term subtracts half of the weight of the disconnected sea-quark loop associated with photon couplings to the disconnected loop. Similarly, for the d -quark contribution, we obtain a value of $\frac{1}{2}(2d_n - \frac{2}{3}d_n + d_p)$, and under charge symmetry, the two light quark contributions become equal,

$$\langle \Lambda^* | \hat{\mu}_\ell^{\text{conn}} | \Lambda^* \rangle = \frac{1}{2} \left(2u_p - \frac{2}{3}u_p + u_n \right). \quad (12)$$

To test the $\bar{K}N$ model prediction of Eq. (12), we draw on the same set of configurations explored in Ref. [10], where the left-hand side of the equation, $\langle \Lambda^* | \hat{\mu}_\ell^{\text{conn}} | \Lambda^* \rangle$, was calculated. These calculations are based on the $32^3 \times 64$ full-QCD ensembles created by the PACS-CS collaboration [1], made available through the International Lattice Data Grid (ILDG) [71]. The ensembles provide a lattice volume of $(2.9 \text{ fm})^3$ with five different masses for the light u and d quarks, and constant strange-quark simulation parameters. We simulate the valence strange quark with a hopping parameter of $\kappa_s = 0.13665$, reproducing the correct kaon mass in the physical limit [72]. We use the squared pion mass as a renormalization group invariant measure of the quark mass. The scale is set via the Sommer parameter [73] with $r_0 = 0.492 \text{ fm}$ [1]. The nucleon magnetic form factors are determined on these lattices using the methods introduced in Ref. [68] and refined in Ref. [70], providing values of $u_p = 1.216(17) \mu_N$ and $u_n = -0.366(19) \mu_N$ at the lightest pion mass. Results are reported for the lowest nontrivial momentum transfer of $Q^2 \simeq 0.16 \text{ GeV}^2/c^2$.

Lattice QCD results from Ref. [10] for the light- and strange-quark magnetic form factors of the $\Lambda(1405)$ are plotted as a function of pion mass in Fig. 3. As mentioned earlier, the flavor symmetry present at heavy quark masses is broken as the u and d masses approach the physical point, where the strange magnetic form factor drops to nearly zero. The light quark sector contribution differs significantly from the molecular $\bar{K}N$ model prediction until the lightest quark mass is reached. At this point, the direct matrix element calculation, $\langle \Lambda^* | \hat{\mu}_\ell^{\text{conn}} | \Lambda^* \rangle$ of Ref. [10], “ $\Lambda(1405)$ light sector” in Fig. 3, agrees with the prediction of the “connected $\bar{K}N$ model” developed here and summarized in Eq. (12). This agreement confirms that the $\Lambda(1405)$ observed in lattice QCD at quark masses resembling those of Nature is dominated by a molecular $\bar{K}N$ structure. At the lightest pion mass, the light-quark magnetic form factor of the $\Lambda(1405)$ is [10]

$$\langle \Lambda^* | \hat{\mu}_\ell^{\text{conn}}(Q^2) | \Lambda^* \rangle = 0.58(5) \mu_N, \quad (13)$$

at $Q^2 \simeq 0.16 \text{ GeV}^2/c^2$. The connected $\bar{K}N$ model of Eq. (12) predicts

$$\langle \Lambda^* | \hat{\mu}_\ell^{\text{conn}}(Q^2) | \Lambda^* \rangle = 0.63(2) \mu_N. \quad (14)$$

It is important to note that the shift in the prediction due to the omission of photon couplings to the disconnected sea-quark

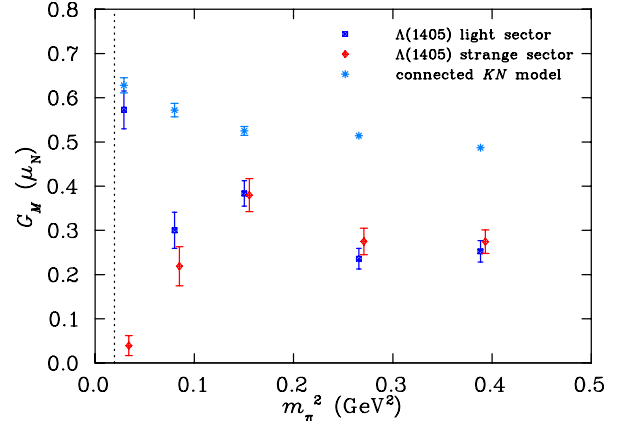


FIG. 3. The light (u or d) and strange quark contributions to the magnetic form factor of the $\Lambda(1405)$ at $Q^2 \simeq 0.16 \text{ GeV}^2/c^2$ from Ref. [10] are presented as a function of the light u - and d -quark masses, indicated by the squared pion mass, m_π^2 . Sector contributions are for single quarks of unit charge. The lattice calculations are compared to the predictions of the connected $\bar{K}N$ model developed herein and summarized in Eq. (12). The vertical dashed line indicates the physical pion mass. The strange form factor results are offset a small amount from the light sector in the m_π^2 -axis for clarity.

loop is significant. In the case where such couplings are included, the prediction of the $\bar{K}N$ model is significantly larger at $\langle \Lambda^* | \hat{\mu}_\ell | \Lambda^* \rangle = (2u_p + u_n)/2 = 1.03(2) \mu_N$. Thus, it is important for the lattice community to continue to work towards a determination of these disconnected-loop contributions, particularly for resonances where coupled channel dynamics play an important role.

The light-quark sector contributions to the magnetic form factor of the $\Lambda(1405)$ calculated in lattice QCD [10] have been examined in the context of a molecular $\bar{K}N$ model in which the quark-flow connected contributions to the magnetic form factor have been identified. This enables a quantitative analysis of the extent to which the light-quark contributions are consistent with a molecular bound-state description.

Identification and removal of the quark-flow disconnected contributions to the $\bar{K}N$ model have been made possible via a recently developed graded-symmetry approach [44]. It is interesting to note that the relative contribution of connected to disconnected contributions is in the ratio 1 : 2 for both flavor-singlet and flavor-octet representations of the Λ baryon.

Using new results for the magnetic form factors of the nucleon at a near-physical quark mass of $m_\pi = 156 \text{ MeV}$, the connected $\bar{K}N$ model predicts a light-quark sector contribution to the $\Lambda(1405)$ of $0.63(2) \mu_N$, which agrees remarkably well with the direct calculation of $0.58(5) \mu_N$ from Ref. [10]. This confirms that the internal structure of the $\Lambda(1405)$ is dominated by a $\bar{K}N$ molecule.

The $\Lambda(1405)$ observed in lattice QCD has significant overlap with local three-quark operators and displays a dispersion relation consistent with that of a single baryon. This implies that the $\bar{K}N$ bound state is localised. Furthermore, it is striking that the nucleon maintains its properties so well when bound.

Future work will focus on the isolation of nearby multi-

particle scattering states in the finite lattice volume, and explore their quark sector contributions to the magnetic form factors. Using the formalism developed in Ref. [74] one can then combine all the low-lying contributions observed in lattice QCD and make contact with the full resonance structure.

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